

# Asymmetric Stochastic Volatility in Nordic Stock Markets

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## Abstract

The goal of this paper is to investigate the asymmetric impact of innovations on volatility and the relationship between the stock return and volatility dynamics in the case of Nordic stock markets using the framework of asymmetric stochastic volatility models. The empirical findings provide strong evidence of asymmetry, significant and high volatility persistence in the stock markets of the Nordic region. The most interesting and different results obtained from the present paper are that there are both of a low variability of volatility and a high volatility persistence in the stock markets of Denmark, Finland, Norway and Sweden. Additionally, the stock markets of the Denmark, Finland, Norway and Sweden, which have leverage effect, have lower variability of volatility and in these markets the future volatility is relatively certain.

**Keywords:** Asymmetric Stochastic Volatility, MCMC, Nordic Stock Markets

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## 1. Introduction

The volatility of asset returns is still one of the major issues of financial econometrics. The understanding of volatility in stock markets matters because volatility has an important role in option pricing, portfolio management and asset allocation. It is well known that stock market volatility, defined as the conditional variance or standard deviation of stock returns, changes over time.

The relationship between a stock market index (or a stock price) and its volatility has been studied widely in market economies and also it is well documented that a negative shock increases the stock market volatility more than the positive shock at the same magnitude and this circumstance is described as “asymmetry”. As mentioned in Cappiello et al. (2006), asymmetric volatility could be handled in two ways: “leverage effect” and “volatility feedback effect”. The leverage effect is described as the notion that a fall in stock price causes an increase in the debt-equity ratio (financial leverage) of the firm and the risk (volatility) of the firm increases right after (Selçuk, 2005). On the other hand, volatility feedback effect is specified as the notion that once volatility is priced, an expected increase in volatility enhances the required return on equity, leading to an urgent stock price downfall. The main difference between leverage effect and volatility feedback effect is the direction of causality between stock returns and volatility. In the leverage effect, the direction of causality is running from the stock returns to volatility whilst the volatility feedback effect implies that the causality running from the volatility to stock returns.

The main goal of the present paper is to investigate the asymmetric impact of innovations on volatility and the relationship between the stock return and volatility dynamics in the case of Nordic markets, which have been researched far less than the other markets, using univariate asymmetric stochastic volatility approach. Although stock return and its volatility in advanced markets have been well studied, there exist relatively few contributions to return and volatility dynamics in Nordic region.

The volatility in the Nordic stock markets tends to be relatively higher in comparison with advanced stock markets. Also, the trading volume and the number of traded firms of the Nordic stock markets are narrow but are growing at a greater rate than advanced markets.

The present paper differs from the extant literature in the following way: to the best of our knowledge, it is the first study that examines the stock returns and volatility dynamics and asymmetric innovations to volatility in the stock markets from the Nordic region using the framework of the asymmetric stochastic volatility models.

The remainder of the paper is organized as follows: in section 2 we discuss the literature review and in section 3 we present the econometric methodology. Section 4 contains the data description and empirical results of the study. The 5<sup>th</sup> and last section includes conclusions.

## 2. Literature Review

As mentioned above, there exist fewer empirical studies on the Nordic region and the empirical findings on asymmetric volatility are conflicted. Booth et al. (1997) investigated the volatility in the stock markets of Nordic region, i.e. Denmark, Finland, Norway and Sweden using EGARCH model and they reached the findings on the persistency of volatility and also findings on asymmetric impact of innovation on volatility for the Nordic stock markets.

Kulp-Tag (2007a) also explored the volatility in the stock markets of Nordic region, i.e. Denmark, Finland, Norway and Sweden using EGARCH model and reported that negative

innovations have a higher impact on volatility than positive innovations, so the asymmetric volatility exists in Nordic stock markets.

Kulp-Tag (2007b) examined the volatility in the stock markets of Nordic region, i.e. Denmark, Finland, Norway and Sweden using GARCH model and its extensions and found out weak evidence of asymmetric volatility in Nordic stock markets.

Urooj et al. (2009) used ARCH and GARCH models in their empirical study and found out the persistency of volatility and volatility clustering in the stock market of Finland.

### 3. Econometric Methodology

A general representation of a volatility model for a stationary series of returns  $r_t$  takes the form:

$$\begin{aligned} r_t &= \mu_t + y_t \\ y_t &= \sigma_t \varepsilon_t \end{aligned} \quad (1)$$

where  $\varepsilon_t$  is i.i.d. random variable with zero mean and unit variance and  $\sigma_t$  is either a deterministic or stochastic random process which depends on the past values of returns.  $\mu_t$  denotes either a constant or an autoregressive variable with a parameter close to zero and  $y_t$  denotes a stochastic process i.e. demeaned returns.

The main issue is related to which process  $\sigma_t$  follows; if  $\sigma_t$  is expressed as a deterministic function of lagged (squared) returns, we are within the ARCH models (Engle, 1982; Bollerslev, 1986), which have achieved widespread popularity in applied empirical research (Pellegrini and Rodriguez, 2007). On the other hand, when  $\sigma_t$  is expressed as a stochastic function of an unobserved latent variable, we introduce the stochastic volatility model proposed by Taylor (1986). Stochastic volatility models are attractive because they are close to the models often used in Financial Theory to represent the behaviour of financial prices and their statistical properties are easy to derive using well-known results on log-normal distributions (Broto and Luiz, 2004). The main relative advantages of stochastic volatility models are discussed by Carnero et al. (2004) and also Das et al. (2011) emphasise that stochastic volatility models have the capability to provide one-step-ahead prediction and to better harmonise with excess kurtosis and leverage effects compared to GARCH models.

The stochastic volatility model is represented as the following form;

$$\begin{aligned} y_t &= \exp(h_t/2) \varepsilon_t \\ h_t &= \gamma + \phi h_{t-1} + \eta_t \end{aligned} \quad (2)$$

where  $h_t$  is latent stochastic volatility which equals to  $\ln \sigma_t^2$ .  $\varepsilon_t$  is i.i.d. random variable with zero mean and unit variance and also  $\eta_t$  is i.i.d. random variable with zero mean and variance  $\sigma_\eta^2$ , independent of  $\varepsilon_t$ .  $\sigma_\eta^2$  indicates the volatility (variability) of volatility and measures the uncertainty about future volatility. The parameter  $\phi$  is described as a measure of the persistence of shocks to the volatility. There is such a trade-off relationship between  $\sigma_\eta^2$  and  $\phi$ ; namely when  $\phi$  approximates to one,  $\sigma_\eta^2$  tends to approximate to zero.

As mentioned Ghysels et. al. (1996), it can be noticed that if  $\varepsilon_t$  and  $\eta_t$  are allowed to be correlated with each other, the model can pick up the kind of asymmetric behaviour. Indeed a negative correlation between  $\varepsilon_t$  and  $\eta_t$  induces a leverage effect. Harvey and Shephard (1996), propose a specification which considers contemporaneous dependence and allows the correlation between  $\varepsilon_t$  and  $\eta_t$  as  $corr(\varepsilon_t, \eta_t) = \rho$ . In other respects, Jacquier et. al. (2004) propose a specification which considers intertemporal dependence and allows the correlation between  $\varepsilon_t$  and  $\eta_{t-1}$  as  $corr(\varepsilon_t, \eta_{t-1}) = \rho$ .

Asai and McAleer (2005) present a specification that captures asymmetry in “dynamic leverage” model through the direct negative correlation between returns and volatility innovations as the following form:

$$\begin{aligned} y_t &= \exp(h_t/2) \varepsilon_t \\ h_{t+1} &= \mu + \phi h_t + \eta_t \quad \varepsilon_t \sim N(0,1) \quad \eta_t \sim N(0, \sigma_\eta^2) \\ E(\varepsilon_t \eta_t) &= \rho \sigma_\eta \end{aligned} \quad (3)$$

We could describe this type of asymmetry, namely when  $\rho < 0$ , as the Dynamic Leverage Stochastic Volatility model. When  $\rho = 0$ , there exists no dynamic leverage between the innovations to returns and volatility (Asai and McAleer, 2005).

#### 4. Data and Empirical Results

The data set involves daily closing price indices of four Nordic countries for the period from January 2, 2009 to September 30, 2016 (post-2008 Global financial crisis period) and consists of stock indices of Denmark (KFX Index), Finland (HEX Index), Norway (OBX Index) and Sweden (OMX Index). The source of data is the Bloomberg Database. Finally, we calculate the stock returns from the stock market indices of the selected countries using the  $\ln(P_t/P_{t-1}) \times 100$  formula where  $P_t$  denotes the value of the stock price indices of each country at time  $t$ .

The descriptive statistics for the stock returns of each stock market indices are reported in Table 1.

**Table 1:** Descriptive Statistics of the Stock Returns

	Denmark	Finland	Norway	Sweden
<b>Mean</b>	0.07202	0.03162	0.05988	0.04659
<b>Median</b>	0.09882	0.05341	0.07564	0.06063
<b>Maximum</b>	6.36103	7.39165	7.45211	6.43374
<b>Minimum</b>	-6.55617	-7.83603	-6.71443	-8.42423
<b>Std. Dev.</b>	1.26198	1.36927	1.45405	1.34654
<b>Skewness</b>	-0.01181	-0.07234	-0.08221	-0.16030
<b>Kurtosis</b>	5.15024	5.54899	5.24386	5.75986
<b>Jarque-Bera</b>	372.43	529.07	410.23	625.61
<b>Probability</b>	0.00000	0.00000	0.00000	0.00000
<b>Observations</b>	1933	1948	1945	1945

All of the stock return series have small mean and the standard deviations of the stock returns are greater than the means of stock returns, indicating that the stock market of Nordic countries follow a random walk process. Additionally, the stock return series have negative skewness and the excess kurtosis for each is significantly positive, indicating that they have heavy tails relative to the normal distribution, which is also typical in these financial data (Ding and Vo, 2012).

The stochastic volatility models can be estimated using different techniques. The most popular approaches are the quasi-maximum likelihood method as proposed by Harvey and Shephard (1996) and the Markov Chain Monte Carlo (MCMC) method which was introduced by Jacquier et. al. (1994). In this study, we employ the MCMC approach for estimating Dynamic Leverage model and we use the code provided by Yasuhiro Omori<sup>(1)</sup> utilized for the WinBUGS software. In MCMC estimation strategy, we determine the prior values as  $\mu \sim \text{Inverse-Normal}(-10,1)$ ,  $\rho \sim \text{Inverse-Uniform}(-1,1)$ ,  $\sigma_{\eta}^2 \sim \text{Inverse-Gamma}(2.5,0.025)$  and  $\phi \sim \text{Inverse-Beta}(20,1.5)$  following Yasuhiro Omori and MCMC sampler is also initialized by setting the values  $\mu = -9$ ,  $\sigma_{\eta}^2 = 100$ ,  $\phi = 0.95$  and  $\rho = -0.4$  following also Yasuhiro Omori. We obtain the posterior means of the coefficients ignoring the first 10.000 iterations and utilizing the following 90.000 iterations in all cases. The posterior means of parameter estimates with 95% posterior credibility intervals are presented in Table 2.

The estimation results shown in Table 2 indicate that the volatility persistence coefficients  $\phi$  are in between 0.939 (Denmark) and 0.987 (Norway) and the empirical findings imply that there exists a remarkable volatility persistence and strong evidence of volatility clustering in the Nordic stock markets.

**Table 2:** The Estimation Results of the Posterior Means of Parameters

	$\hat{\mu}$	$\hat{\phi}$	$\hat{\rho}$	$\hat{\sigma}_{\eta}$
<b>Denmark</b>	-8.935*	0.939*	-0.514*	0.263*
	(0.097)	(0.013)	(0.059)	(0.031)
	[-9.121 -8.738]	[0.910 0.962]	[-0.620 -0.389]	[0.207 0.329]
<b>Finland</b>	-8.843*	0.978*	-0.669*	0.164*
	(0.134)	(0.006)	(0.065)	(0.021)
	[-9.105 -8.577]	[0.964 0.988]	[-0.784 -0.534]	[0.126 0.208]
<b>Norway</b>	-8.725*	0.987*	-0.779*	0.169*
	(0.194)	(0.004)	(0.050)	(0.016)
	[-9.100 -8.331]	[0.979 0.993]	[-0.861 -0.670]	[0.139 0.203]
<b>Sweden</b>	-8.898*	0.973*	-0.744*	0.205*
	(0.130)	(0.006)	(0.046)	(0.023)
	[-9.149 -8.633]	[0.961 0.985]	[-0.833 -0.651]	[0.165 0.254]

**Notes:** The posterior standard deviations and 95% posterior credibility intervals are presented in the parentheses and brackets, respectively. \* denotes statistical significance at the 5% level.

The posterior means of the coefficient  $\hat{\rho}$ , indicating the correlation between innovations to returns and volatility, are negative and statistically significant at the 5% level for all of Nordic

stock markets. The smallest value is -0.514 for Denmark and the highest value is -0.779 for Norway. It can be concluded that there is a high leverage effect in the stock markets of Denmark, Finland, Norway and Sweden so a negative shock increases the stock market volatility more than the positive shock at the same magnitude in these markets.

In other respects, the posterior means of the volatility of volatility coefficient  $\hat{\sigma}_\eta$ , indicating the measure of uncertainty about the future volatility are within the range of 0.164 (Finland) and 0.263 (Denmark). It can be concluded that the Nordic stock markets exhibit a low variability of volatility and also the future volatility is relatively certain in these stock markets.

## 5. Conclusions

The present paper is a first attempt to find out stock market volatility dynamics in the Nordic economies using the framework of asymmetric stochastic volatility models. The data set involves daily closing price indices of four Nordic countries and consists of stock indices of Denmark (KFX Index), Finland (HEX Index), Norway (OBX Index) and Sweden (OMX Index).

The empirical findings provide strong evidence of asymmetry for all of the Nordic stock markets. The estimation results display that there is a high leverage effect in the stock markets of Denmark, Finland, Norway and Sweden so a negative shock increases stock market volatility more than a positive shock at the same magnitude in these markets.

Also it is shown that the Nordic stock markets have significant and high volatility persistence. It can be implied that the volatility clustering occurs in the stock markets of Denmark, Finland, Norway and Sweden. Moreover, the empirical results demonstrate that there exists a low variability of the volatility in stock markets of the Nordic region which are Denmark, Finland, Norway and Sweden.

The most interesting and unique results obtained from the present paper are that there is both a low variability of volatility and a high volatility persistence in the stock markets of Denmark, Finland, Norway and Sweden. Additionally, we can conclude that the stock markets of Denmark, Finland, Norway and Sweden, which have leverage effect, have lower variability of volatility and in these markets the future volatility is relatively certain. The results presented in this paper could be a guide for the investors who are planning to invest in stock markets from the Nordic region.

## Notes

(1) Code used in MCMC estimations of the parameters can be downloaded from Professor Yasuhiro Omori's web site <http://www.omori.e.u-tokyo.ac.jp/WinBUGS/index.htm>

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