Migration, Congestion and Growth[1]

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Abstract

This article analyzes the effect of migration from a less advanced economy to a more advanced economy on economic growth. The analysis is performed in a two-country growth model with endogenous fertility, in which congestion diseconomies are incorporated. The model shows that out-migration increases fertility and reduces human capital in the source economy. At the same time, in-migration reduces fertility and can increase or decrease the average level of human capital in the host economy. I show how migration affects the inter-temporal evolution of human capital in the world economy. I also demonstrate that a tax imposed on immigrants in the host economy can increase human capital accumulation in the receiving and sending economies and the world as a whole.

Keywords: Migration, congestion diseconomies, fertility, human capital, growth, brain drain, brain dilution tax

JEL Codes: D3, F22, J1, O0

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1. Introduction

The number of international migrants increased by more than twofold during the last five decades and reached 211 million in 2010. In high-income OECD countries the share of foreign-born in the population almost tripled between 1960 and 2010 and now is above 10 percent in most of the countries.[3] Akin to international migration, internal migration has also been on the rise. Thus, in 2011 there were 250 million internal migrant workers in China alone.[4] Similarly, tens of millions of people have migrated within the boundaries of the United States, European Union and Russia. Large flows of internal (and international) migration have also been recorded in Latin America, Asia and Africa. All over the world, the pattern of internal and international migration is similar: most people migrate from poorer, less developed regions and countries to wealthier, more developed regions and countries.

These flows of migration have also largely been considerably spatially concentrated. Thus, for international migration, it has been broadly argued that in most countries immigrants tend to be largely concentrated in particular areas, especially within major metropolitan agglomerations.[5] Similarly, a concentration of emigrants in particular areas within the source countries is a central element behind the theory of migration networks.[6] For internal migration, where people typically move from smaller urban and semi-urban localities to larger urban centers, spatial concentration of the migration flows is even more evident.

This article studies one of the classical questions in development economics: How does migration from a less advanced region or country to a more advanced region or country affect economic growth? The analysis is performed in a two-country growth model with endogenous fertility in the tradition of Galor and Tsiddon (1997) building on Azarnert (2010a).[7] In this context, the particular contribution of the present study is to introduce "congestion diseconomies" – the concept that plays an important role in the urban economics literature, but

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3 Over the same period, the proportion of immigrants originating from developing countries in the population of high-income OECD countries increased from 1.5 to 8 percent.

4 In 2012, National Bureau of Statistic of China reported that out of 252.78 million migrant workers in China in 2011, migrant workers who left their home provinces accounted for 158.63 million and migrant workers who worked within their home provinces reached 94.15 million.

5 See, for instance, Bartel (1989) and Borjas (1998, 1999) for evidence from the United States, Edin et al. (2003) for Scandinavia, and Stark (1991) for a more general picture.

6 See, for example, Durand et al. (2001) and Hanson (2006) for evidence on spatial pattern of Mexican emigration to the United Sates and McKenzie and Rapoport (2010) for some references to the theory of migration networks.

7 Other works in this context that the present model is connected to include Dahan and Tsiddon (1998), Morand (1999), Galor and Moav (2000, 2002), Moav (2005), Galor and Mountford (2006, 2008), Azarnert (2008, 2016), and Strulik and Weisdorf (2014).

up to the present was neglected by the voluminous literature on endogenous fertility, human capital and growth. (For surveys of the recent literature, see Galor (2011, 2012).) This extension is called for to properly integrate the consequences of population concentration in modern urbanized world into the long run dynamics of the models of endogenous economic growth.

A negative effect of the geographical concentration of economic agents in smaller, densely populated areas within economy has long been well recognized in the urban and regional economics and the new economic geography. Thus, for example, according to Henderson (2002), United Nations data (UNCHS) indicate that, typically around the world, moving from a city of 250,000 to one of 2.5 million is associated empirically with an 80 percent increase in commuting times and housing rental prices. Richardson (1987) argues from data for four developing countries that moving from a small city to a megacity raises per capita investment costs per family in urban infrastructure by threefold. Detailed arguments can be found, for instance, in Kanemoto (1980) and Fujita (1989).

This negative effect referred to as "congestion diseconomies", or "congestion costs" has been exploited in a large number of theoretical contributions, such as, for instance, Tabuchi (1998), Duranton and Puga (2004), Sato and Yamamoto (2005), Venables (2005), Henderson and Wang (2005), Sato (2007), Henderson and Venables (2009), among others. To model congestion, researchers usually assume that all production in a city occurs in a Central Business District (CBD). Surrounding the CBD there is a circle of residences, owned by absentee landlords, where each worker occupies one unit of land. This creates a per worker cost of urban living, which consists of the cost of commuting to the CBD and land rent. The commuting costs and the land rent are considered to increase with population concentration, which, in turn, reduces the workers' disposable time and income. The present work adopts this approach.[8]

In this model, an increase in population density is associated with a reduction in fertility. Evidence supporting such a connection abounds (e.g. Murphy et al. (2008), Simon and Tamura (2009); cf. also Malmberg (2012) where further reference can be found). In theoretical literature with congestion diseconomies a negative effect of population concentration on fertility has been shown in Sato and Yamamoto (2005) and Sato (2007). However, these two models abstract

[8] The positive effect of population concentration (i.e. “agglomeration economies”) is captured in the present model by the assumption that the wage per efficiency unit of labor in the more developed host economy is higher than that in the less developed source economy. Assuming that an increase in the population density increases the efficiency in the provision of human capital will further depress human capital formation in the less developed economy and encourage human capital formation in the more developed economy leaving the qualitative nature of this paper’s major result unaltered.

from investment in education and therefore do not consider human capital accumulation, which is the major theme of the present study.

This paper's major contribution is to the literature on migration and brain drain. From 1970s onward, the classical brain drain literature has considered international migration as a detrimental factor to the development of poor countries. This literature has argued that the level of human capital in developing countries is growing slowly because the developed countries "siphon off" their highly educated workers, thus increasing the productivity of developed world at the expense of the developing countries (Bhagwati and Wilson, 1989).

This view has recently been challenged in a range of theoretical publications, such as, for example, Mountford (1997), Stark et al. (1998), Beine et al. (2001), Stark and Wang (2002), Fan and Stark (2007), Mountford and Rapoport (2011), among others. This new literature has argued that the possibility of migration to a higher wage foreign country raises the return to education, thus leading to an increase in human capital formation, which can outweigh the negative effect of brain drain in the source economies. Within this context, it has also been argued that, although a brain gain may happen in the short run, relaxation of restrictions on the emigration of high-skilled workers will damage the economic growth a source country in the long run (Chen, 2006). In Azarnert (2012), I demonstrate that the possibility of a low-skilled employment in a higher wage foreign country lowers the relative attractiveness of acquiring human capital thus further reducing the number of skilled workers and the aggregate level of human capital in the source economy.[9]

The present paper abstracts from the possibility of a beneficial brain drain and follows the classical approach in brain drain literature that a less developed economy open to out-migration loses its higher-skilled workers, as has been broadly established for both international (Miyagiwa, 1991) and internal (Zhang, 2002) types of migration.[10] To generate a positive self-

[9] Khrainche (2015) evaluates the optimal duration of a temporary worker permit from the point of view of the host country.

[10] Within this context, Beine et al. (2008) argue that among sending countries there appear to be more losers from brain drain than winners. Moreover, several case studies suggest that, in contrast to the beneficial brain drain hypothesis, the possibility of migration to a more developed economy, or, similarly, a more developed region within economy may reduce the incentive for acquiring human capital, as, for example, has been broadly found in the context of Mexican migration to the USA (e.g. Kandel and Kao, 2001; McKenzie and Rapoport, 2011; Antman, 2011) and rural-urban migration in China (de Brauw and Giles, 2016). For a general criticism of the beneficial brain drain hypothesis see Schiff (2005). For an assessment of the magnitude, intensity and determinants of the brain drain along with a review of the literature that amassed during four decades of economic research see Docquier and Rapoport (2012). De la Croix and Docquier (2012) argue that for the majority of developing countries some brain drain is inevitable, as a corollary of poverty.

selection among the migrants, the present model refers to the cost of migration.[11] This allows us to derive the threshold level of human capital that divides the source economy's population into two groups: the more skilled, for whom migration is optimal, and the less skilled, for whom migration is not worthwhile.[12]

In this model, brain drain migration always reduces the total (and average) human capital stock in the less developed source economy. At the same time, the effect of migration on the host economy's average level of human capital is uncertain. Thus, it can either be positive, if skilled emigrants from the less developed economy possess on average more human capital than the local agents in the more advanced economy, or negative, if the new-comers are on average less skilled than the indigenous population. Therefore, the analysis shows that, even if immigrants' skills are fully transferable across the boundaries, and hence migration involves no losses of human capital, nonetheless, it can lead to a reduction in the average levels of human capital in both the sending and receiving economies.[13] I derive the exact condition that guarantees an increase/ decrease in the host society's average human capital level as a result of immigration. I also show how migration affects the inter-temporal evolution of human capital in the world economy. I derive conditions for migration that takes place in one period of time to increase or decrease the average level of human capital in the next period and further on.

In this work, I also analyze the effect of migration on the levels of individuals' utility. As their optimal choice, migration always increases utility of the migrants themselves. The effect of migration on individuals who do not migrate is, however, uncertain. Thus, for individuals who remain in the less advanced economy, the out-migration of the agents with superior skills generates two conflicting effects. First, it decreases the average level of human capital in the society, thereby reducing the return on investment in their offspring's education, which, in turn, reduces the resulting children's human capital stock. Second, it also reduces population concentration in the source economy. As follows from the resulting reduction in

[11] The cost of migration can be substantial. Thus, for example, for the US interstate migration, Bayer and Juesson (2012) obtain a cost estimate close to US$ 35,000 or roughly two thirds of an average annual household income.

[12] The aforementioned literature on brain drain with brain gain typically assumes that all agents in the less advanced economy would want to migrate, but that migration is possible only for an exogenously given fraction of skilled agents. Zhang (2002) argues that there exists a unique level of human capital that makes migration worthwhile for agents with human capital above it, but does not compute this threshold explicitly.

[13] The negative effect of immigration on the host society's average human capital is consistent, for example, with the situation in the United States, where, after a reversal in the quality of immigration in the 1980s, immigrants on average are less skilled than the US natives (Borjas, 1999). Similarly, immigrants from the former republics of the Old Soviet Union to Russia also typically possess less human capital than the Russian natives.

congestion diseconomies, this increases the remaining individuals' disposable time available for work and parenting, thereby increasing the adult agents' consumption along with the number of their children. Following the same idea, immigration increases population concentration in the receiving economy. As a consequence, the resulting increase in the cost of congestion reduces the indigenous agents' disposable time and hence decreases their consumption and fertility. At the same time, immigration from the less developed economy may increase or decrease the average level of human capital in the more developed economy, thereby increasing or decreasing the local agents' offspring's human capital stock via a human capital externality. I derive the exact conditions for migration to increase or decrease utility among individuals who do not migrate in the sending and receiving economies. I also consider a public policy intervention to help mitigate a possible negative effect of migration.

The idea of an income tax paid by highly skilled emigrants to compensate their home-country society for the negative externality imposed by their out-migration on those left behind has been actively debated since 1970s, when it was originally proposed by Jagdish Bhagwati. (See, for example, McHale (2009) and Docquier and Rapoport (2012), where some further references can be found.)

In this work, I extend the discussion toward a tax imposed on immigrants to compensate the indigenous population in the host economy for the in-migration-driven negative externality. By analogy with the brain drain, or Bhagwati tax, this type of redistribution can be referred to as a “brain dilution tax”. This type of taxation is close in spirit to the current practices in several countries, in particular, in Europe, where a legal status is provided to foreign investors.[14] I present and analyze the effect of such tax on human capital accumulation in the receiving and sending economies and the world as a whole. The analysis demonstrates that for the receiving economy, the effect of taxation is twofold: First, the tax increases the threshold level of human capital, above which migration is worthwhile, which reduces the size of immigration and increases the average quality of the immigrants. Second, redistribution of the proceeds to the local agents stimulates growth of the high skilled population in the more advanced economy. At the same time, for the source economy, a tax imposed on the immigrants in the host economy decreases the out-migration-driven reduction in the average level of human capital in the source economy, thereby encouraging its economic growth. Moreover, if the out-migration of the

[14] Within this context, a notorious example is the United Kingdom, where passports are "sold" for 1 million British pounds.

agents with superior skills reduces the levels of utility among individuals who remain in the poor source economy, the brain dilution tax can have a positive effect on their utility as well.[15]

The remainder of the article is organized as follows. Section 2 develops the basic model and analyzes the consequences of migration for the sending and receiving economies and the world as a whole. Section 3 introduces the brain dilution tax and explores its effect on human capital accumulation. Section 4 considers an incomplete assimilation of the migrants' offspring in the host economy. Section 5 concludes.

2. The Basic Structure of the Model

Consider an overlapping-generations economy, in which activity extends over an infinite discrete time. In every period the economy produces a single homogenous good using a constant-returns-to-scale technology with human capital as the only input. In each generation, agents live for two periods: childhood and adulthood. During childhood, individuals acquire human capital. During adulthood, they work, become parents and bring up their offspring. As parents, adult individuals allocate a positive fraction of their time to feeding and raising their children and invest in the education of their children.

Suppose the world that consists of two entities: a more advanced, more developed economy denoted by MD, and a less advanced, less developed economy denoted by LD. For some exogenous reason, in the more developed economy wages and the average level of human capital are higher than those in the less developed economy.

To specify the pattern of migration, suppose that in the beginning of the second period of life, young adult individuals from the poorer less developed economy can migrate to the richer more developed economy. If young adults migrate, they work, become parents, bring up and educate their children at their destination.

2.1. Congestion diseconomies, wages and the cost of migration

Suppose that in any economy i ($i = MD, LD$), an adult individual incurs the basic cost of living that includes land rent, i.e., the costs of living space rented from the absentee landlords, who keep their rental revenues "outside the model", and commuting costs to the business district for

[15] In their study of comparative economic development, Ashraf et al. (2010) argue that geographic isolation (i.e., in the context of the present paper, higher migration costs) has generated a persistent beneficial effect on the process of development and contributed to the contemporary variation in the standards of living across countries.

work and shopping and to the recreational area for relaxation. These costs represent congestion diseconomies, as is commonly assumed in the urban economics literature (e.g. Kanemoto (1980), Fujita (1989), Tabuchi (1998), Duranton and Puga (2004), Sato and Yamamoto (2005), Venables (2005), Henderson and Wang (2005), Sato (2007), Henderson and Venables (2009), among others). Furthermore, as consistent with the aforementioned urban economics literature, the costs of congestion diseconomies is assumed to increase with population density, which implies that an increase in the population size in any region reduces each individual's disposable time and income in that economy.

More specifically, the basic cost of living, or congestion diseconomies, is measured here in terms of work (or parenting) time forgone at a_t^i per individual living in economy i ($i = MD, LD$) at time t, where $a_t^i \in [a^{i,\min}, a^{i,\max}]$ is a positive, strictly increasing function of the population size in that economy, L_t^i:

$$a_t^i = a^i(L_t^i), \tag{1}$$

where $\forall L_t^i \geq 0$, $a^i(L_t^i) \geq 0$, $a^{i'}(L_t^i) > 0$, $\lim_{L_t^i \to 0} a^i(L_t^i) = a^{i,\min}$, $\lim_{L_t^i \to \infty} a^i(L_t^i) = a^{i,\max}$, $0 \leq a^{i,\min} < a^{i,\max} < 1$.

In anticipation of the further discussion of migration the following is assumed:

A1. *The wage per efficiency unit of labor in the more developed economy,* w^{MD}, *is higher than the wage per efficiency unit of labor in the less developed economy,* w^{LD}, *and the wage differential is large enough, so that in period* t, $w^{MD}(1-a_t^{MD}) > (1-a_t^{LD})w^{LD}$.

This assumption reflects the fact that, in general, the net return to human capital, i.e., the workers' real wages in the richer more developed regions and, especially, countries are higher than in the poorer less developed regions and countries. In either economy, the wage per efficiency unit of labor, w^i, is fixed over time, as follows from, for instance, the assumption of a CRS technology with a single factor of production.

To complete the description of the world economy, we also suppose that, to migrate to the more developed economy, a young adult individual born in the less developed economy must pay the amount m that covers the costs of migration. I also assume that migration involves no losses of human capital, so that, after migration to the more developed economy, any unit of human capital acquired in the less developed economy prior to migration is as productive, as the locally acquired skills.[16]

Since by construction in this model, only migration from the poorer less developed economy to the richer more developed economy is worthwhile, in the world economy there are potentially three types of individuals: (1) MD, individuals born in the more developed economy, who always remain in the economy where they were born, (2) LD, individuals born in the less developed economy who remain in the economy where they were born, (3) M, individuals born in the less developed economy who migrate to the more developed economy. The conditions that lead to the decision to migrate are analyzed below in Section 2.5.

2.2. The Formation of Human Capital

In any period t, an adult of type j ($j = MD, LD, M$) born in economy i ($i = MD, LD$) is characterized by a skill level h_t that is distributed according to the cumulative density function $F_t^i(\cdot)$ over the strictly positive support $[h_t^{i,\min}, h_t^{i,\max}]$. It is assumed throughout that in period t, the average level of human capital in the more developed economy is higher than that in the less developed economy; $\bar{h}_t^{MD} > \bar{h}_t^{LD}$.

In each period of life individuals are endowed with one unit of time. In the first period, children devote their entire time for the acquisition of human capital. The acquired human capital increases if their time investment is supplemented with real resources invested in their education.

The human capital level of a child, who becomes an adult in period $t+1$, depends on the parental real expenditure on the child's education, e_t^j, and on the average level of human capital of all adult individuals residing in economy i in period t, which is defined as $\bar{h}_t^i = \int h_t dF_t^i(h^i),\ i = MD, LD$, according to the human capital production function or learning technology is described by

$$h_{t+1} = \Theta(e_t^j, \bar{h}_t^i). \qquad (2)$$

This learning technology captures an external spillover effect that arises from the average society's level of human capital, $\bar{h}_t$. Such formulation is consistent with the so-called global or atmospheric externality, which implies that an increase in the average level of human capital in the society as a whole increases the rate of return on investment in human capital for

16 Assuming that the migrants' skills are not fully transferable will strengthen this work's major insight.

the children's generation. First introduced by Tamura (1991), the assumption that the average level of human capital in society is an input in the production of human capital for each individual became common in the literature. This externality has been utilized, e.g. by Tamura (1996), Galor and Tsiddon (1997), Morand (1999), Viaene and Zilcha (2002), de la Croix and Doepke (2003), Henderson and Wang (2005), Azarnert (2008, 2009, 2010a), among many others.

A particular form of human capital production function is specified below in equation (9).

2.3. *The Optimization of Parents*

Agents of any type derive utility from their own consumption in adulthood and from the net total future income of their children in the economy where the children were born.[17] The utility function of an individual of any type j ($j = MD, LD, M$) born at time $t-1$ is therefore

$$U_t^j = (1-\beta)\log C_t^j + \beta\log(I_{t+1}^{N,j}), \tag{3}$$

where C_t^j is an individual's own consumption, $I_{t+1}^{N,j}$ is the net future income of that individual's offspring and $\beta \in (0,1)$ captures the relative weight given to children.[18]

In every period *t*, adult individuals are endowed with one unit of time. From this unit the basic costs of living associated with congestion diseconomies measured in terms of work time foregone at a_t^i per individual living in economy i, are deducted. The remainder of their time adults allocate between childbearing and labor force participation. In either economy, the cost of feeding and raising children is measured in terms of work time foregone at δ per child. The cost of acquiring human capital in any economy i is measured in units of the wage per efficiency unit of labor in that economy, w^i.

Individuals are assumed to behave as atomistic agents, so that the migrants neglect the effect of their migration on the basic cost of living, a_t^i, and the average level of human capital, $\bar{h}_t^i$, in the economies of their origin and destination, Similarly, all agents neglect the effect of

[17] This assumption rules out the situation when parents, who do not find it worthwhile to migrate to the MD region, will consider the possibility of migration for their offspring. This anticipates the further assumption (Section 2.9) that migration is possible in period t only. Assuming that parents take the possibility of their offspring's migration into account will change the threshold level of human capital, above which agents migrate, without altering the qualitative nature of this paper's results.

[18] In the utility function postulated by Galor and Moav (2002) individuals differ with respect to the relative weight given to the quality of their children. In the utility function used in Azarnert (2010b), where the choice is between giving birth to one child per parent or remaining childless, individuals differ with respect to the weight given to the child.

their decision with respect to the number of their offspring on the basic cost of living and the average level of human capital in the children's generation.

To maximize utility, an adult of any type j simultaneously chooses a current consumption, C_t^j, the number of children, N_t^j, and invests e_t^j units of w^i in each child's education subject to the following budget constraint:[19]

$$C_t^j + w^i(\delta h_t + e_t^j)N_t^j \leq w^i h_t(1-a_t^i), \text{ if } j = i = MD, LD,$$

$$C_t^j + w^i(\delta h_t + e_t^j)N_t^j \leq w^i h_t(1-a_t^i) - m, \text{ if } j = M \text{ and } i = MD. \quad (4)$$

The right-hand side of equation (4) represents an adult's income net of the costs associated with congestion diseconomies in the economy where that individual lives, a_t^i, which is allocated between consumption and the total cost of rearing children.

The total potential future income of the individual's offspring net of the basic costs associated with congestion diseconomies in either economy in children's generation, a_t^i, is:

$$I_{t+1}^{N,j} = N_t^j h_{t+1} w^i (1-a_{t+1}^i), \quad j = i = MD, LD. \quad (5)$$

2.4. Quantity - Quality Tradeoff

From optimization, an adult's consumption is

$$C_t^j = (1-\beta)\begin{cases} h_t w^i(1-a_t^i), & \text{if } j = i = MD, LD \\ h_t w^i(1-a_t^i) - m, & \text{if } j = M \text{ and } i = MD. \end{cases} \quad (6)$$

That is, a fraction $1-\beta$ of an adult's net full income is devoted to consumption and hence a fraction β is devoted to childrearing.

In order to allocate resources between children's quantity and quality, an adult makes two simultaneous decisions. First, he decides how much consumption to forego during his adulthood to rear a family. Second, he decides what amount of resources to invest in the education of his children to increase their skill level.

For an individual of any type in the case of a non-corner solution, the standard condition of setting the marginal rate of substitution between quality and quantity equal to the price implies that

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[19] The time constraint requires that $0 \leq 1 - a_t^i - h(\delta + e_t^j / h_t)N_t^j \leq 1 - a_t^i$, if $j = i = MD, LD$, and $0 \leq 1 - a_t^i - (m / w^i h_t) - (\delta + e_t^j / h_t)N_t^j \leq 1 - a_t^i$, if $j = M$ and $i = MD$.

$$\frac{h_{t+1}}{N_t^j} - \frac{\delta h_t + e_t^j}{N_t^j \big/ \left(dh_{t+1} / de_t^j \right)} = 0 \quad \text{if} \quad e_t^j > 0, \tag{7}$$

where h_{t+1} / N_t^j is the marginal rate of substitution between quality and quantity, $w^i(\delta h_t + e_t^j)$ is the cost of an additional child for a given level of parental investment in the child's education and $w^i N_t^j / [dh_{t+1} / de_t^j]$ is the marginal cost of children's quality (human capital) for a given number of children.

From equation (7), optimization with respect to child's quality thus implies that

$$h_{t+1} = \left(\delta h_t + e_t^j \right) \frac{dh_{t+1}}{de_t^j}. \tag{8}$$

The next subsection discusses the solution for the parents' optimization problem for a particular form of the human capital production function.

2.5. Choice of Fertility and Investment in Education

To characterize optimal choices of fertility and investment in education, suppose that in either economy all children born in this economy have access to the same technology of human capital production:

$$h_{t+1} = (\mu + e_t^j)^\gamma \bar{h}_t^i, \quad 0 < \gamma < 1, \ 0 < \mu < 1, \ \text{where } i = MD, LD \text{ and } j = MD, LD, M. \tag{9}$$

This learning technology implies that children of the migrants from the LD economy born in the MD economy become similar to the indigenous population of the MD economy. I relax this assumption below in Section (4).

Given (9), the optimal choice of investment in the children's education of an individual of any type in either economy is[20]

$$e_t^j = \frac{\gamma \delta h_t - \mu}{1 - \gamma}, \quad j = LD, MD, M. \tag{10}$$

so that, according to (10),

$$h_{t+1} = \left(\frac{\gamma}{1-\gamma} (\delta h_t - \mu) \right)^\gamma \bar{h}_t^i, \quad i = LD, MD. \tag{11}$$

Given the amount of resources allocated to children's education, the desired fertility is

[20] An assumption that $h_t^{i,\min} > \mu / \gamma\delta$ ensures that all parents invest in the education of their children.

$$N_t^j = \begin{cases} \frac{\beta(1-\gamma)}{\delta-(\mu/h_t)}(1-a_t^i), & \text{if } \ j = i = MD, LD \\ \frac{\beta(1-\gamma)}{\delta-(\mu/h_t)}(1-a_t^i - \frac{m}{w^i h_t}), & \text{if } \ j = M \text{ and } \ i = MD. \end{cases} \tag{12}$$

Equation (10) shows that the optimal choice of investment in the offspring's education and hence the children's human capital levels (Eq. 11) is positively related to the parent's human capital, although parental human capital does not enter the learning technology directly. Equation (12) displays the traditional negative relationship between the parental level of human capital and the choice of fertility.

In the next section, I derive conditions that lead to the decision to migrate.

2.6. The Decision to Migrate

To characterize individuals' choice with respect to migration, recall that by definition in this model the average level of human capital in the more developed economy is higher than that in the less developed economy; $\bar{h}_t^{MD} > \bar{h}_t^{LD}$. This implies that, for any given fraction of the parental income invested in the offspring's education, $e_t w^i$, the human capital production function (9) yields a higher level of the child's human capital in the more developed economy. Similarly, assumption A1 implies that the net wage per unit of efficiency labor in the more developed economy is also higher than that in the less developed economy; $w^{MD}(1-a_t^{MD}) > w^{LD}(1-a_t^{LD})$. As a consequence, in the world economy under consideration only one way of migration from the LD economy to the MD economy is worthwhile. At the same time, a migrant should pay the cost of migration, m. Therefore, a young adult individuals from the LD economy decide to migrate if their utility in the case of migration is higher than their utility in the LD economy; $U_t^M > U_t^{LD}$.

Substituting an individual's own consumption along with that individual's offspring's net potential income in the case of migration, as shown in equations (4) and (5), respectively ($j = M$), into the utility function (3) and comparing the result to the level of utility in the case of no migration ($j = LD$) allows us to compute the following threshold level of human capital:[21]

[21] To derive the threshold, note that $U_t^M > U_t^{LD}$ if the following condition holds:

$(1-\beta)\log((1-\beta)w^{MD}h_t(1-a_t^{MD}) - m)$

$$\hat{h}_t = \frac{m}{w^{MD}(1-a_t^{MD}) - w^{LD}(1-a_t^{LD})\left(\dfrac{\bar{h}_t^{LD}(1-a_{t+1}^{LD})}{\bar{h}_t^{MD}(1-a_{t+1}^{MD})}\right)^{\beta}}. \tag{13}$$

Notice that, since the negative denominator of equation (13) implies that the other way of migration – from the MD to the LD economy – is worthwhile, which is ruled out by definition in this model, this threshold is either positive or meaningless. Therefore, the further analysis is performed under assumption that in the threshold equation the denominator is positive.[22]

This threshold level divides the LD population into two groups:

1. *Individuals with human capital levels lower than the threshold* $\hat{h}_t$ who choose to remain in the LD economy.
2. *Individuals with human capital levels greater than the threshold* $\hat{h}_t$ who choose to migrate to the MD economy.

2.7. The Effect of Migration on Individuals' Consumption, Fertility and Per-Child Human Capital Levels

To establish the effect of migration on individual agents' optimal choice of their own consumption at adulthood, the number of children and the offspring' per-child human capital levels, compare the results of optimization in the case of migration to those that are obtained in the absence of migration. For comparison, denote the case of migration by the superscript M and the case of no migration by the superscript NM.

From equation (6), subtracting an adult individual's level of consumption in the case of migration (C_t^M) from that in the absence of migration (C_t^{NM}), one can obtain that for the indigenous population in the MD economy, as well as for individuals who remain in the LD economy

$$\Delta C_t^{j,M} \equiv C_t^{j,M} - C_t^{j,NM} = (1-\beta)w^i h_t(a_t^{i,NM} - a_t^{i,M}), \quad j = i = MD, LD. \tag{14}$$

$$+\beta\log\left(\frac{\beta(1-\gamma)}{\delta-(\mu/h_t)}(1-a_t^{MD}-\frac{m}{w^{MD}h_t})(\frac{\gamma}{1-\gamma}(\delta h_t-\mu))^{\gamma}\bar{h}_t^{MD}(1-a_{t+1}^{MD})w^{MD}\right)$$

$$> (1-\beta)\log((1-\beta)w^{LD}h_t(1-a_t^{LD})) + \beta\log\left(\frac{\beta(1-\gamma)}{\delta-(\mu/h_t)}(1-a_t^{LD})(\frac{\gamma}{1-\gamma}(\delta h_t-\mu))^{\gamma}\bar{h}_t^{LD}(1-a_{t+1}^{LD})w^{LD}\right).$$

[22] As follows from assumption A1, $w^{MD}(1-a_t^{MD}) > w^{LD}(1-a_t^{LD})$. Therefore, to ensure that the denominator is positive it is enough to assume that $\bar{h}_t^{MD} \geq \bar{h}_t^{LD}(1-a_{t+1}^{LD})/(1-a_{t+1}^{MD})$.

Similarly, from equation (12),

$$\Delta N_t^{j,M} \equiv N_t^{j,M} - N_t^{j,NM} = \frac{\beta(1-\gamma)}{\delta-(\mu/h_t)}(a_t^{i,NM} - a_t^{i,M}), \quad j = i = MD, LD\,. \tag{15}$$

Migration from the LD economy to the MD economy increases the congestion costs in the MD economy, while reducing these costs in the LD economy, so that in period t, $a_t^{MD,M} > a_t^{MD,NM}$ and $a_t^{LD,M} < a_t^{MD,NM}$. Therefore, from equations (13) and (14), it is evident that migration reduces fertility and adults' consumption in indigenous population in the MD economy, while increasing fertility and consumption among individuals who remain in LD economy.

From equation (11),

$$\Delta h_{t+1}^{j,M} \equiv h_{t+1}^{j,M} - h_{t+1}^{j,NM} = \left(\frac{\gamma}{1-\gamma}(\delta h_t - \mu)\right)^{\gamma}(\bar{h}_t^{i,M} - \bar{h}_t^{i,NM}), \quad j = i = MD, LD\,. \tag{16}$$

Since, as established in Section 2.6, for agents with human capital levels above the threshold $\hat{h}_t$ it is worthwhile to leave the LD economy, migration always reduces the average level of human capital in this economy, $\bar{h}_t^{LD,NM} > \bar{h}_t^{LD,M}$. As a consequence, in the case of the out-migration from this economy, the per-child human capital levels of the offspring of the agents who remain in the LD economy decline. As to the offspring of individuals born in the MD economy, their per-capita human capital levels can either increase, if the average level of in-migrants' human capital is higher than the average level of human capital of the indigenous population, $\bar{h}_t^{M} > \bar{h}_t^{MD}$, or decrease, if, on the average, the migrants are less skilled than the indigenous population, $\bar{h}_t^{M} < \bar{h}_t^{MD}$. The exact condition for $\bar{h}_t^{M}$ to be higher or lower than $\bar{h}_t^{MD}$ is established below in equations (A3) and (A4) in Appendix A.

Proceeding to the migrants themselves, from equation (6),

$$\Delta C_t^{M} \equiv C_t^{M} - C_t^{LD} = (1-\beta)h_t((w^{MD}(1-a_t^{MD,M}) - (m/h_t)) - w^{LD}(1-a_t^{LD,M}))\,. \tag{17}$$

This allows us to conclude that among migrants consumption increases among individuals with human capital levels above the threshold $\hat{h}_t'$:

$$\hat{h}_t' = \frac{m}{w^{MD}(1-a_t^{MD}) - w^{LD}(1-a_t^{LD})} \tag{18}$$

and decreases among individuals with human capital levels below this threshold.[23]

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[23] Note that if $\bar{h}_t^{MD} \geq \bar{h}_t^{LD}(1-a_{t+1}^{LD})/(1-a_{t+1}^{MD})$, $\hat{h}_t' > \hat{h}_t$ and, therefore, consumption among immigrants with human capital levels that fall in the range of $[\hat{h}_t, \hat{h}_t'[$ is lower in the case of migration. For these

Following the same steps as in the case of the MD and LD groups, in the case of the migrants,

$$\Delta N_t^M \equiv N_t^M - N_t^{LD} = \frac{\beta(1-\gamma)}{\delta - (\mu/h_t)}(a_t^{i,NM} - a_t^{i,M} - (m/w^{MD}h_t)) \tag{19}$$

and

$$\Delta h_{t+1}^M \equiv h_{t+1}^{MD,M} - h_{t+1}^{LD,NM} = \left(\frac{\gamma}{1-\gamma}(\delta h_t - \mu)\right)^{\gamma}(\bar{h}_t^{MD,M} - \bar{h}_t^{LD,M}), \tag{20}$$

which implies that migration reduces their fertility, while increasing their children's human capital levels.

Note also that for all three groups of population, the effect of migration on the adults' consumption and their children's per-capita human capital stock is stronger the higher is the agent's level of human capital. In contrast, in the case of fertility, the effect of migration weakens with the agents' human capital.

2.8. The Effect of Migration on Individuals' Utility

As their optimal choice, migration always increases the level of utility of the individuals who decide to migrate, so that $U_t^M > U_t^{LD}$. It however can either increase or decrease the levels of utility for the other two groups in population.

Thus, for individuals who remain in the LD economy, the level of utility can increase if the negative effect of the decline in the average level of human capital in their economy caused by the out-migration of the agents with superior skills is not sufficiently large, so as to outweigh the positive effect of the corresponding reduction in congestion. Therefore, comparing the corresponding levels of utility with and without migration, $U_t^{LD,M} > U_t^{LD,NM}$, and thereby individuals who remain in the LD economy are better off with the out-migration if

$$\frac{\bar{h}_t^{LD,NM}}{\bar{h}_t^{LD,M}} < \left(\frac{1-a_t^{LD,M}}{1-a_t^{LD,NM}}\right)^{\frac{1}{\beta}} \frac{1-a_{t+1}^{LD,M}}{1-a_{t+1}^{LD,NM}}. \tag{21}$$

On the other hand, the level of utility of individuals in the MD economy can increase with migration if and only if the positive effect of the increase in the average level of human capital in this economy owing to the in-migration is sufficiently large, so as to outweigh the

migrants with relatively low human capital the positive effect on their utility through the increase of their children's quality (Eq. 20) outweighs the negative effect through the reduction in their own consumption and the quantity of their offspring.

negative effect of the corresponding increase in congestion. Comparing the levels of utility with and without migration, $U_t^{MD,M} > U_t^{MD,NM}$ if

$$\frac{\bar{h}_t^{MD,M}}{\bar{h}_t^{MD,NM}} > \left(\frac{1-a_t^{MD,NM}}{1-a_t^{MD,M}}\right)^{\frac{1}{\beta}} \frac{1-a_{t+1}^{MD,NM}}{1-a_{t+1}^{MD,M}}. \tag{22}$$

If these conditions do not hold, i.e., inequalities (21) and (22) are reversed, the populations in the origin and host economies can be worse off with free uncontrolled migration.

2.9. The Dynamical System

This section analyzes the dynamic behavior of the society's average level of human capital. To characterize the effect of migration on the inter-temporal evolution of human capital in the world economy, I examine the effect of migration in period t on the average level of human capital in the next period, in which migration is impossible.[24] I consider first the world as a whole. Next, I proceed to the analysis of each of the economies separately.

The average human capital level in period $t+1$ is defined as

$$\bar{h}_{t+1} \equiv \int h_{t+1} dF_{t+1}(h) = \int N_{t+1} h_{t+1} dF_t(h) \Big/ \int N_{t+1} dF_t(h). \tag{23}$$

Distinguishing parents of each type and denoting the world economy by the superscript W, the average level of human capital in the world as a whole in period $t+1$ in the case of migration, $\bar{h}_{t+1}^{W,M}$, is

$$\bar{h}_{t+1}^{W,M} = \frac{\int N_t^{MD,M} h_{t+1} dF_t^{MD}(h^{MD}) + \int\limits_{h_t^{LD} > \hat{h}_t} N_t^{M} h_{t+1} dF_t^{LD}(h^{LD}) + \int\limits_{h_t^{LD} \le \hat{h}_t} N_t^{LD,M} h_{t+1} dF_t^{LD}(h^{LD})}{\int N_t^{MD,M} dF_t^{MD}(h^{MD}) + \int\limits_{h_t^{LD} > \hat{h}_t} N_t^{M} dF_t^{LD}(h^{LD}) + \int\limits_{h_t^{LD} \le \hat{h}_t} N_t^{LD,M} dF_t^{LD}(h^{LD})}. \tag{24}$$

Correspondingly, in the absence of migration, the average level of human capital in the world economy, $\bar{h}_{t+1}^{W,NM}$, is

$$\bar{h}_{t+1}^{W,NM} = \frac{\int N_t^{MD,NM} h_{t+1} dF_t^{MD}(h^{MD}) + \int N_t^{LD,NM} h_{t+1} dF_t^{LD}(h^{LD})}{\int N_t^{MD,NM} dF_t^{MD}(h^{MD}) + \int N_t^{LD,NM} dF_t^{LD}(h^{LD})}. \tag{25}$$

[24] As follows from the property of the learning technology (9) with respect to the average level of human capital in the society, the effect of migration on human capital levels in each of the economies evolves further from one generation to the next.

Given the number of children and the levels of human capital investment among the three types of agents, as determined in Section 2.5, the average human capital levels in period $t+1$ in both cases are, respectively,

$$\bar{h}_{t+1}^{W,M} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \Bigg[\bar{h}_t^{MD,M} \Big(\int (1-a_t^{MD,M}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{MD}$$

$$+ \int_{h_t^{LD} > \hat{h}_t} (1-a_t^{MD,M} - (m/w^{MD} h_t)) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \Big)$$

$$+ \bar{h}_t^{LD,M} \int_{h_t^{LD} \le \hat{h}_t} (1-a_t^{LD,M}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \Bigg] \Bigg/ \tag{26}$$

$$\Bigg[\int (1-a_t^{MD,M})(\delta - (\mu/h_t))^{-1} dF_t^{MD} + \int_{h_t^{LD} > \hat{h}_t} (1-a_t^{MD,M} - (m/w^{MD} h_t)(\delta - (\mu/h_t))^{-1} dF_t^{LD}$$

$$+ \int_{h_t^{LD} \le \hat{h}_t} (1-a_t^{LD,M})(\delta - (\mu/h_t))^{-1} dF_t^{LD} \Bigg]$$

and

$$\bar{h}_{t+1}^{W,NM} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \Bigg[\bar{h}_t^{MD,NM} \int (1-a_t^{MD,NM}) h_t (\delta h_t - \mu)^{\gamma-1} \bar{h}_t^{MD,NM} dF_t^{MD}$$

$$+ \bar{h}_t^{LD,NM} \int (1-a_t^{LD,NM}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \Bigg] \Bigg/ \tag{27}$$

$$\Bigg[\int (1-a_t^{MD,NM})(\delta - (\mu/h_t))^{-1} dF_t^{MD} + \int (1-a_t^{LD,NM})(\delta - (\mu/h_t))^{-1} dF_t^{LD} \Bigg].$$

Comparing the levels of human capital in the case of migration ($\bar{h}_{t+1}^{W,M}$) to that in the absence of migration ($\bar{h}_{t+1}^{W,NM}$), as shown above in equations (26) and (27), allows us to determine precisely whether migration increases or decrease the average level of human capital in the world as a whole. Thus, if $\bar{h}_{t+1}^{W,M} > \bar{h}_{t+1}^{W,NM}$, migration increase the world economy's level of human capital. In contrast, if $\bar{h}_{t+1}^{W,M} < \bar{h}_{t+1}^{W,NM}$, the average level of human capital in the world is higher in the absence of migration.

Proceeding now to the analysis of each of the economies separately, let us recall that by construction in this model migration always decreases the level of human capital at the origin. Therefore, for $\bar{h}_{t+1}^{W,M} > \bar{h}_{t+1}^{W,NM}$, a migration-driven positive effect on the average level of human capital in the MD economy in period $t+1$ should necessarily be large enough, so as to outweigh its negative effect on the LD economy. On the other hand, if $\bar{h}_{t+1}^{W,M} < \bar{h}_{t+1}^{W,NM}$, the host economy's average level of human capital in period $t+1$ can increase with migration, if $\bar{h}_{t+1}^{MD,M} > \bar{h}_{t+1}^{MD,NM}$, or decrease if $\bar{h}_{t+1}^{MD,M} < \bar{h}_{t+1}^{MD,NM}$. The exact condition is relegated to Appendix A.

Moreover, although in this framework migration does not alter the total amount of human capital in the world in the period when it takes place, nonetheless it can lead to a reduction in the average human capital levels in both the sending and receiving economies in this period. Thus, if on the average, the migrants are less skilled than the relatively highly skilled indigenous population in the destination economy, $\bar{h}_t^{M} < \bar{h}_t^{MD,NM}$, their arrival can reduce the average level of human capital in the host economy as well. The exact condition for $\bar{h}_t^{MD,M}$ to be higher or lower than $\bar{h}_t^{MD,NM}$ is established in equations (A3) and (A4) in Appendix A.

3. Public Policy Intervention

As shown previously in Section 2.8, free uncontrolled migration can lead to a reduction in the utility of the populations in the migrants' origin and destination economies. Similarly, as shown in Section 2.9, from the children's generation on, it can be conductive to a decline in the average per-capita human capital levels not only in the source economy, but also in the host economy and in the world as a whole. In this case, an income redistribution financed by taxes levied on the migrants, who by definition always gain from migration, can help mitigate the negative effect of their migration.

In this Section, I consider a tax imposed on immigrants to compensate the indigenous population in the host economy for the in-migration-driven negative externality. By analogy with the brain drain, or Bhagwati tax, this type of redistribution can be referred to as a "brain dilution tax". I present and analyze the effect of this tax on human capital accumulation in the destination economy and in the world as a whole. I also shed some light on the effect of taxation in one economy on human capital accumulation (and utility) in the other.

To specify the tax-transfer scheme, suppose that in period t in the host economy there is one common lump-sum tax, T_t, levied on any immigrant with the proceeds redistributed as a lump-sum subsidy, S_t, to any indigenous adult individual born in this economy.[25] This scheme yields that the brain dilution tax imposed on an immigrant in period t is

$$T_t = \frac{\int S_t N_t^{MD} dF_{t-1}^{MD}(h^{MD})}{\int_{h_t^{LD} > \hat{h}_t} N_t^{LD} dF_{t-1}^{LD}(h^{LD})} , \tag{28}$$

where the number of immigrants $\int_{h_t^{LD} > \hat{h}_t} N_{t-1}^{LD} dF_{t-1}^{LD}(h^{LD})$ is either positive, or meaningless.

With the tax-transfer scheme specified above, following the same steps as in Section 2.5, the number of children in the indigenous population in the MD economy increases to

$$N_t^{MD,M} = \frac{\beta(1-\gamma)}{\delta - (\mu/h_t)}(1 - a_t^{MD,M} + \frac{S_t}{w^{MD} h_t}) \tag{29}$$

and among immigrants declines to

$$N_t^{M} = \frac{\beta(1-\gamma)}{\delta - (\mu/h_t)}(1 - a_t^{MD,M} - \frac{m + T_t}{w^{MD} h_t}) , \tag{30}$$

while the per-child investment in children's education in both groups of agents remains unaffected as shown in equation (10).

[25] The standard brain drain tax, as proposed by Bhagwati and followers, is usually modeled as an income tax paid by emigrants on top of their regular income tax. However, in a model with endogenous fertility and explicit quantity-quality tradeoff a labor-income tax appears to be inferior to a lump-sum tax, because taxation of labor income increases the relative cost of child quality, thus reducing the parental per-child investment in the education of the tax-payers' offspring. Similarly, if the proceeds are distributed proportionally to the labor income of the local agents, this increases the relative cost of the quantity of their children, which further reduces their optimal fertility choice, thereby amplifying the negative effect of immigration on the size of the local population in the MD economy.

Within the framework of the present model, a labor-income tax at the rate τ_t imposed on immigrants reduces their per-child educational investment to

$$e_t^M = \frac{\gamma\delta h_t(1-\tau_t) - \mu}{1-\gamma} ,$$

while a subsidy at the rate s_t to the local agents' labor income reduces the number of their children to

$$N_t^{MD} = \frac{\beta(1-\gamma)}{\delta - (\mu/((1+s_t)h_t)}(1 - a_t^{MD,M}) .$$

Therefore, in this case, denoted by the superscript K, following the same steps as in Section 2.8, the average level of human capital in the MD economy in period $t+1$ is

$$\bar{h}_{t+1}^{MD,MK} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \left[\bar{h}_t^{MD,MK} \left(\int (1-a_t^{MD,MK} + \frac{S_t}{w^{MD} h_t}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{MD} \right.\right.$$

$$\left.\left. + \int_{h_t^{LD} > \hat{h}_t^K} (1-a_t^{MD,MK} - \frac{m+T_t}{w^{MD} h_t}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \right) \right] \Bigg/ \tag{31}$$

$$\left[\int (1-a_t^{MD,MK} + \frac{S_t}{w^{MD} h_t})(\delta - (\mu/h_t))^{-1} dF_t^{MD} + \right.$$

$$\left. \int_{h_t^{LD} \le \hat{h}_t^K} (1-a_t^{LD,MK} - \frac{m+T_t}{w^{MD} h_t})(\delta - (\mu/h_t))^{-1} dF_t^{LD} \right],$$

while the average level of human capital in the world economy as a whole is

$$\bar{h}_{t+1}^{W,MK} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \left[\bar{h}_t^{MD,MK} \left(\int (1-a_t^{MD,M} + \frac{S_t}{w^{MD} h_t}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{MD} \right.\right.$$

$$\left. + \int_{h_t^{LD} > \hat{h}_t^K} (1-a_t^{MD,MK} - \frac{m+T_t}{w^{MD} h_t}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \right)$$

$$\left. + \bar{h}_t^{LD,M} \int_{h_t^{LD} \le \hat{h}_t^K} (1-a_t^{LD,M}) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \right] \Bigg/ \tag{32}$$

$$\left[\int (1-a_t^{MD,MK} + \frac{S_t}{w^{MD} h_t})(\delta - (\mu/h_t)^{-1} dF_t^{MD} \right.$$

$$\left. + \int_{h_t^{LD} > \hat{h}_t^K} (1-a_t^{MD,MK} - \frac{m+T_t}{w^{MD} h_t})(\delta - (\mu/h_t))^{-1} dF_t^{LD} + \int_{h_t^{LD} \le \hat{h}_t^K} (1-a_t^{LD,MK})(\delta - (\mu/h_t))^{-1} dF_t^{LD} \right],$$

where

$$\hat{h}_t^K = \frac{m + T_t}{w^{MD}(1-a_t^{MD,MK}) - w^{LD}(1-a_t^{LD,MK})\left(\frac{\bar{h}_t^{LD,MK}(1-a_{t+1}^{LD,MK})}{\bar{h}_t^{MD,MK}(1-a_{t+1}^{MD,MK})}\right)^{\beta}} \tag{33}$$

and

$$T_t^K = \frac{\int S_t(1-a_{t-1}^{MD})(\delta - (\mu/h_{t-1}))^{-1} dF_{t-1}^{MD}}{\int\limits_{h_t^{LD} > \hat{h}_t^k} (1-a_{t-1}^{LD})(\delta - (\mu/h_{t-1}))^{-1} dF_{t-1}^{LD}} \ . \tag{34}$$

Comparing the average levels of human capital in the MD economy and the world as a whole, as shown above in equations (31) and (32), respectively, to those in the absence of migration, as shown in equations (A1) in Appendix and (27), the brain dilution tax can be set in such a manner, so that $\bar{h}_{t+1}^{MD,MK} \geq \bar{h}_{t+1}^{MD,NM}$ and $\bar{h}_{t+1}^{W,MK} \geq \bar{h}_{t+1}^{W,NM}$.[26]

Similarly, the amount of taxation can be set so as to fully neutralize the negative effect of in-migration on the local agents' consumption and fertility.[27] With redistribution it is also possible to achieve the same level of utility for the local agents in the host economy in period t, when migration takes place, as in the absence of migration, or even make them better off with migration. The average level of human capital in the world economy with the subsidy that equalizes the levels of utility of the adult agents in the MD economy with and without migration is shown in Appendix B.

The reciprocal effect of the redistribution in one economy on human capital accumulation in the other is also worthy of mention. Thus, for example, the brain dilution tax imposed on the immigrants in the host economy increases the human capital threshold, which discourages migration of individuals, whose human capital levels fall in the range of $]\hat{h}_t, \hat{h}_t^K]$. This, in turn, decreases the out-migration-driven reduction in the average level of human capital in the source economy, thereby encouraging its economic growth. Likewise, an exit tax imposed on the emigrants in the source economy also increases that threshold in a similar way. This makes migration undesirable for potential migrants with lower levels of human capital, thus increasing the average level of human capital of the migrants who arrive at the MD economy.

[26] If taxation is relatively high, it is also possible that there will be no migration in the equilibrium and, thus, the average level of human capital in the world will be exactly the same as shown in equation (27). Arithmetically, this happens when $h_t^{LD,\max} \leq \hat{h}_t^K$.

[27] In this particular case, the necessary amount of the subsidy, so that with the subsidy $C_t^{MD,MK} = C_t^{MD,NM}$ and $N_t^{MD,MK} = N_t^{MD,NM}$, is $S_t = w^{MD} h_t (a_t^{MD,MK} - a_t^{MD,NM})$.

Following the same intuition, if migration is associated with a reduction in the levels of utility among individuals who do not migrate, a tax on the migrants in one economy can have a positive effect on the local agents' utility in the other.

4. Incomplete Assimilation

In previous sections we assumed that the migrants fully assimilate in their host economy, and that their children born in the MD economy becomes similar to that economy's indigenous population. It has however been broadly argued in the literature that the human capital of an individual's ethno-cultural group is an important input in the formation of that person's own human capital (e.g. Borjas (1992, 1995), among others).

To consider an incomplete assimilation of the migrants' offspring in their host economy, the human capital production function (9) can be re-formulated, so as to allow for a partial dependence of their human capital levels on the average level of human capital at their parent's origin:[28]

$$h_{t+1} = (\mu + e_t^j)^\gamma (\psi \bar{h}_t^{MD} + (1-\psi)\bar{h}_t^{LD}), \quad 0 \le \psi \le 1, \quad 0 < \gamma < 1, \; 0 < \mu < 1. \tag{38}$$

It this case, denoted by the superscript A, a human capital threshold that makes migration worthwhile becomes

$$\hat{h}_t^A = \frac{m}{w^{MD}(1-a_t^{MD}) - w^{LD}(1-a_t^{LD})\left(\dfrac{\bar{h}_t^{LD}(1-a_{t+1}^{LD})}{(\psi \bar{h}_t^{MD} + (1-\psi)\bar{h}_t^{LD}(1-a_{t+1}^{MD})}\right)^\beta}. \tag{39}$$

Note that for any $\psi \in (0,1)$, $\hat{h}_t^A > \hat{h}_t$, as specified in equation (13), which discourages migration of individuals, whose human capital levels fall in the range of $]\hat{h}_t, \hat{h}_t^A]$.

This allows us to postulate that an incomplete assimilation of the migrants and their locally born offspring in the more developed host economy has a positive effect on the human-capital-based growth in the less developed source economy. On the other hand, for the more developed host economy, an incomplete assimilation generates two conflicting effects. First, it is associated with an increase in the levels of the migrants' human capital. Second, it reduces the levels of human capital of the migrants' offspring, which can have negative consequences for the human-capital-based growth in the host economy in the future.

[28] Bisin and Verdier (2011) provide an extensive review of the literature on the transmission of cultural, ethnic and religious traits across generations.

5. Conclusion

This article analyzes the effect of migration from a less developed economy to a more developed economy on economic growth. The analysis is performed in the context of a growth model with endogenous fertility, in which congestion diseconomies are incorporated. The model shows that out-migration increases fertility and reduces the total (and average) human capital stock in the source economy. At the same time, immigration reduces fertility and can either increase or decrease the average level of human capital in the host economy. I derive a condition that determines precisely whether immigration increases or decreases the average level of human capital in the receiving economy. I show how migration affects the inter-temporal evolution of human capital levels in the world economy. I also demonstrate that a tax imposed on the immigrants in the host economy can increase human capital accumulation in the destination and source economies and the world as a whole. Moreover, this tax can also have a positive effect on the utility of individuals who remain in the poor source economy.

Appendix A. *Average levels of human capital in the MD economy with and without free uncontrolled in-migration*

Following the same steps as in Section 2.9, the average level of human capital in the MD economy in period $t+1$ in the absence of migration is

$$\bar{h}_{t+1}^{MD,NM} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \bar{h}_t^{MD,NM} \int h_t(\delta h_t - \mu)^{\gamma-1} dF_t^{MD} \Big/ \int (\delta - (\mu/h_t))^{-1} \bar{h}_t^{MD,NM} dF_t^{MD} \qquad \text{(A1)}$$

while the corresponding average level of human capital in presence of migration is

$$\bar{h}_{t+1}^{MD,M} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \Bigg[\bar{h}_t^{MD,M} \Big(\int (1-a_t^{MD,M}) h_t(\delta h_t - \mu)^{\gamma-1} dF_t^{MD}$$

$$+ \int_{h_t^{LD} > \hat{h}_t} (1-a_t^{MD,M} - (m/w^{MD} h_t)) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \Big) \Bigg] \Bigg/$$

$$\left[\int (1-a_t^{MD,M})(\delta - (\mu/h_t))^{-1} dF_t^{MD} + \int_{h_t^{LD} > \hat{h}_t} (1-a_t^{LD,M} - (m/w^{MD} h_t))(\delta - (\mu/h_t))^{-1} dF_t^{LD} \right] \qquad \text{(A2)}$$

Comparing the level of human capital in the case of migration, $\bar{h}_{t+1}^{MD,M}$, to that in the absence of migration, $\bar{h}_{t+1}^{MD,NM}$, as shown above in equations (A2) and (A1), respectively, allows us to determine precisely whether migration increases or decreases the average level of human capital in the MD economy in period $t+1$.

Similarly, in period t, $\bar{h}_t^{MD,M}$ is higher or lower than $\bar{h}_t^{MD,NM}$ if the average level of the immigrants' human capital, $\bar{h}_t^{M}$, as shown below in Eq. (A3), is higher or lower than the average level of the indigenous population, $\bar{h}_t^{MD}$, as shown in Eq. (A4), correspondingly:

$$\bar{h}_t^{M} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \bar{h}_{t-1}^{LD} \int\limits_{h_t^{LD} > \hat{h}_t} h_{t-1}(\delta h_{t-1} - \mu)^{\gamma-1} dF_{t-1}^{LD} \Bigg/ \int\limits_{h_t^{LD} > \hat{h}_t} (\delta - (\mu/h_t))^{-1} dF_{t-1}^{LD} \qquad \text{(A3)}$$

and

$$\bar{h}_t^{MD} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \bar{h}_{t-1}^{MD} \int h_{t-1}(\delta h_{t-1} - \mu)^{\gamma-1} dF_{t-1}^{MD} \Bigg/ \int (\delta - (\mu/h_{t-1}))^{-1} \bar{h}_{t-1}^{MD} dF_{t-1}^{MD} \ . \qquad \text{(A4)}$$

Appendix B. *The average levels of human capital in the world economy with the subsidy that equalizes* $U_t^{MD,M}$ and $U_t^{MD,NM}$

To compute the amount of the subsidy that is necessary to neutralize the negative effect of migration on the utility of individuals in the host economy, compare the levels of their utility in the case with migration and subsidy and in the absence of migration. With the lump-sum subsidy, S_t, given to the adult agents in the MD economy, $U_t^{MD,M} = U_t^{MD,NM}$ if

$$S_t = \left[(1-a_t^{MD,NM}) \left(\frac{\bar{h}_t^{MD,NM}(1-a_{t+1}^{MD,NM})}{\bar{h}_t^{MD,M}(1-a_{t+1}^{MD,M})} \right)^{\beta} - (1-a_t^{MD,M}) \right] w^{MD} h_t \ . \qquad \text{(A5)}$$

In this case, denoted by the superscript 1, the average level of human capital in the world economy as a whole in period $t+1$ is

$$\bar{h}_{t+1}^{W,M1} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} \left[\bar{h}_t^{MD,M1} \left(\int (1-a_t^{MD,M1})(\delta h_t - \mu)^{\gamma-1} \left(\frac{\bar{h}_t^{MD,NM}(1-a_t^{MD,NM})}{\bar{h}_t^{MD,M1}(1-a_t^{MD,M})} \right)^{\beta} h_t dF_t^{MD} \right.\right.$$

$$\left. + \int\limits_{h_t^{LD} > \hat{h}_t^1} (1-a_t^{MD,M1} - ((m+T_t^1)/w^{MD} h_t)) h_t (\delta h_t - \mu)^{\gamma-1} dF_t^{LD} \right)$$

$$+\bar{h}_t^{LD,M1} \int_{h_t^{LD} \le \hat{h}_t^1} (1-a_t^{LD,M1})(\delta h_t - \mu)^{\gamma-1} h_t dF_t^{LD} \Bigg] \Bigg/ \qquad \text{(A6)}$$

$$\Bigg[\int (1-a_t^{MD,M1})(\delta - (\mu/h_t))^{\gamma-1} \left(\frac{\bar{h}_t^{MD,NM}(1-a_t^{MD,NM})}{\bar{h}_t^{MD,M1}(1-a_t^{D,M1})} \right)^{\beta} dF_t^{MD}$$

$$\int_{h_t^{LD} > \hat{h}_t^1} (1-a_t^{MD,M1} - ((m+T_t^1)/w^{MD} h_t) dF_t^{LD} + \int_{h_t^{LD} \le \hat{h}_t^1} (1-a_t^{LD,M1})(\delta - (\mu/h_t))^{-1} dF_t^{LD} \Bigg],$$

where

$$\hat{h}_t^1 = \frac{m+T_t^1}{w^{MD}(1-a_t^{MD,M1}) - w^{LD}(1-a_t^{LD,M1}) \left(\frac{\bar{h}_t^{LD,M1}(1-a_{t+1}^{LD,M1})}{\bar{h}_t^{MD,M1}(1-a_{t+1}^{MD,M1})} \right)^{\beta}}$$

and

$$T_t^1 = \int \left[\frac{1-a_{t-1}^{MD,NM}}{\delta - (\mu/h_t)} \left(\frac{\bar{h}_t^{MD,NM}(1-a_t^{MD,NM})}{\bar{h}_t^{MD,M1}(1-a_t^{MD,M1})} \right)^{\beta} (1-a_t^{MD,NM}) - (1-a_t^{MD,M1}) \right] h_t dF_{t-1}^{MD} w^{MD} \Bigg/$$

$$\int_{h_t^{LD} > \hat{h}_t^1} (1-a_{t-1}^{LD})(\delta - (\mu/h_{t-1}))^{-1} dF_{t-1}^{LD}.$$

The average level of human capital in the destination or source economies separately can easily be computed following the same steps.

References

Antman FM (2010) The intergenerational effect of parental migration on schooling and work: What can we learn from children's time allocations? *Journal of Development Economics* 96, 200–208

Ashraf Q, Galor O, Ozak O (2010) Isolation and development. *Journal of European Economic Association* 8, 401–412

Azarnert LV (2008) Foreign aid, fertility and human capital accumulation. *Economica* 75, 766–781

Azarnert LV (2010a) Free education, fertility and human capital accumulation. *Journal of Population Economics* 23, 449–468

Azarnert LV (2010b) Après nous le déluge: fertility and the intensity of struggle against immigration. *Journal of Population Economics* 23, 1339–1349

Azarnert LV (2012) Guest-worker migration, human capital and fertility. *Review of Development Economics* 16, 318–330

Azarnert LV (2016) Transportation costs and the great divergence. *Macroeconomic Dynamics* 20, 214–228

Bartel AP (1989) Where do the new U.S. immigrants live? *Journal of Labor Economics* 7, 371–391

Bayer C, Juessen F (2012) On the dynamics of interstate migration: Migration costs and self-selection. *Review of Economic Dynamics* 15, 377–401

Bhagwati J, Wilson JD (1989) *Income Taxation and International Mobility*. Cambridge: MIT Press

Beine M, Docquier F, Rapoport H (2001) Brain drain and economic growth: theory and evidence. *Journal of Development Economics* 64, 275–289

Beine M, Docquier F, Rapoport H (2008) Brain drain and human capital formation in developing countries: winners and losers. *Economic Journal* 118, 631–652

Bisin A, Verdier Th (2011) The economics of cultural transmission and socialization. In Benhabib J, Bisin A, Jackson MO (eds.) *Handbook of Social Economics*, North Holland: Elsevier, Vol. 1A, pp. 333–416

Borjas GJ (1992) Ethnic capital and intergenerational mobility. *Quarterly Journal of Economics* 107, 123–150

Borjas GJ (1995) Ethnicity, neighborhoods, and human-capital externality. *American Economic Review* 84, 365–390

Borjas GJ (1998) To ghetto or not to ghetto? Ethnicity and residential segregation. *Journal of Urban Economics* 44, 228–253

Borjas GJ (1999) *Heaven's Door – Immigration Policy and the American Economy*. Princeton, NJ: Princeton University Press

Chen H-J (2006) International migration and economic growth: a source country perspective. *Journal of Population Economics* 19, 725–748

Dahan M, Tsiddon D (1998) Demographic transition, income distribution and economic growth. *Journal of Economic Growth* 3, 29–52

de Brauw A, Giles J (2016) Migrant opportunity and the educational attainment in rural China. *Journal of Human Resources*. Forthcoming.

de la Croix D, Doepke M (2003) Inequality and growth: why differential fertility matters. *American Economic Review* 93, 1091–1113

de la Croix D, Docquier F (2012) Do brain drain and poverty result from coordination failures? *Journal of Economic Growth* 17, 1–26

Docquier F, Rapoport H (2012) Globalization, brain drain and development. *Journal of Economic Literature* 50, 681–730

Durand J, Massey DS, Zenteno RM (2001) Mexican Immigration to the United States: continuities and changes. *Latin American Research Review* 36, 107–127

Duranton G, Puga D (2004) Micro-foundation of urban agglomeration economies. In Henderson JV, Thisse J-F (eds.) *Handbook of Urban and Regional Economics*, North Holland, Amsterdam: Elsevier, Vol. 4, pp. 2065–2118

Edin P-A, Fredriksson P, Aslund O (2003) Ethnic enclaves and economic success of immigrants – evidence from a natural experiment. *Quarterly Journal of Economics* 118, 329–357

Fan CS, Stark O (2007) International migration and "educated unemployment". *Journal of Development Economics* 83, 76–87

Fujita M (1989) *Urban Economic Theory: Land use and City Size*. Cambridge: Cambridge University Press

Galor O (2011) *Unified Growth Theory*. Princeton University Press

Galor O (2012) The demographic transition: causes and consequences. *Cleometrica* 6, 1–28

Galor O, Moav O (2000) Ability biased technological transition, wage inequality and economic growth. *Quarterly Journal of Economics* 115, 469–498

Galor O, Moav O (2002) Natural selection and the origin of economic growth. *Quarterly Journal of Economics* 117, 1133–1191

Galor O, Mountford A (2006) Trade and the Great Divergence: the family connection. *American Economic Review* 96, 229–303

Galor O, Mountford A (2008) Trading population for productivity: Theory and evidence. *Review of Economic Studies* 75, 1143–1179

Galor O, Tsiddon D (1997) The distribution of human capital and economic growth. *Journal of Economic Growth* 2, 93–124

Hanson G (2006) Illegal migration from Mexico to the United States. *Journal of Economic Literature* 44, 869–924

Henderson JV (2002) Urban primacy, external costs, and quality of life. *Resource and Energy Economics* 24, 95–106

Henderson JV, Wang HG (2005) Aspects of the rural-urban transformation of countries. *Journal of Economic Geography* 5, 23–42

Henderson JV, Venables AJ (2009) The dynamics of city formation. *Review of Economic Dynamics* 12, 233–254

Kandel W, Kao G (2001) The impact of temporary labor migration on Mexican children's educational aspirations and performance. *International Migration Review* 35, 1205–1231

Kanemoto Y (1980) *Theories of Urban Externalities*. North – Holland: Amsterdam

Khraiche M (2015) A macroeconomic analysis of guest worker permits. *Macroeconomic Dynamics* 19, 189–220

Malmberg B (2012) Fertility cycles, age structure and housing demand. *Scottish Journal of Political Economy* 59, 467–482

McHale J (2009) Taxation and skilled Indian migration to the United States: revisiting the Bhagwati tax. Chapter 12 in Bhagwati J, Hanson G (eds.) *Skilled Immigration: Problems, Prospects and Policies*. Oxford University Press

McKenzie D, Rapoport H (2010) Self-selection patterns in Mexico – U.S. migration: the role of migration networks. *Review of Economics and Statistics* 92, 811–821

McKenzie D, Rapoport, H (2011) Can migration reduce educational attainment? Evidence from Mexico. *Journal of Population Economics* 24, 1313–1358

Miyagiwa K (1991) Scale economies in education and the brain drain problem. *International Economic Review* 32, 743–759

Moav O (2005) Cheap children and the persistence of poverty. *Economic Journal* 115, 88–110

Morand OF (1999) Endogenous fertility, income distribution, and growth. *Journal of Economic Growth* 4, 331–349

Mountford A (1997) Can a brain drain be good for growth in the source economy? *Journal of Development Economics* 53, 287–303

Mountford A, Rapoport H (2011) The brain drain and the world distribution of income. *Journal of Development Economics* 95, 4–17

Murphy KM, Simon CJ, Tamura R (2008) Fertility decline, baby boom, and economic growth. *Journal of Human Capital* 2, 262–302

Richardson H (1987) The costs of urbanization: a four country comparison. *Economic Development and Cultural Change* 33, 561–580

Sato Y (2007) Economic geography, fertility and migration. *Journal of Urban Economics* 61, 372–387

Sato Y, Yamamoto K (2005) Population concentration, urbanization, and demographic transition. *Journal of Urban Economics* 58, 45–61

Shiff M (2005) Brain gain: claims about its size and impact on welfare and growth are greatly exaggerated. In Ozden C and Shiff M (eds.) *International Migration, Remittances and Brain Drain*. New York: Palgrave Macmillan, pp. 201–225

Simon CJ, Tamura R (2009) Do higher rents discourage fertility? Evidence from U.S. cites, 1940 – 2000. *Regional Science and Urban Economics* 39, 33–42

Stark O (1991) *The Migration of Labor*. Oxford: Blackwell

Stark O, Helmenstein C, Prskawetz A (1998) Human capital depletion, human capital formation, and migration: a blessing or a "curse"? *Economics Letters* 60, 363–367

Stark O, Wang Y (2002) Inducing human capital formation: migration as a substitution for subsidies. *Journal of Public Economics* 86, 29–46

Strulick H, Weisdorf J (2014) How child costs and survival shaped the industrial revolution and the demographic transition. *Macroeconomic Dynamics* 18, 114 – 144

Tabuchi T (1998) Urban agglomeration and dispersion: a synthesis of Alonso and Krugman. *Journal of Urban Economics* 44, 333–351

Tamura R (1991) Income convergence in an endogenous growth model. *Journal of Political Economy* 99, 522–540

Tamura R (1996) From decay to growth: a demographic transition to economic growth. *Journal of Economic Dynamics and Control* 20, 1237–1261

Venables AJ (2005) Spatial disparities in developing countries: Cities, regions, and international trade. *Journal of Economic Geography* 5, 3–21

Viaene JM, Zilcha I (2002) Capital market integration, growth and income distribution. *European Economic Review* 46, 301–327

Zhang J (2002) Urbanization, population transition, and growth. *Oxford Economic Papers* 54, 91–117